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# Lepton flavor violation in lopsided models and a neutrino mass model

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**Abstract.** A widely adopted theoretical scheme to account for the neutrino oscillation phenomena is the see-saw mechanism together with the "lopsided" mass matrices, which is generally realized in the framework of supersymmetric grand unification. We will show that this scheme leads to large lepton flavor violation at low energy if supersymmetry is broken at the GUT or Planck scale. Especially, the branching ratio of $\mu \to e\gamma$ already exceeds the present experimental limit. We then propose a phenomenological model which can account for the LMA solution to the solar neutrino problem and at the same time predict a branching ratio of $\mu \to e\gamma$ below the present limit.

## 1 See-saw mechanism and "lopsided" structure

The neutrino experiments show that the neutrino parameters have two exotic while interesting features, i.e., the extreme smallness of the neutrino masses and the large size of the neutrino mixing angles [1–3]. According to the recent analyses the atmospheric neutrino oscillation favors the $\nu_\mu$–$\nu_\tau$ process with the best fit values [4]

$$\Delta m^2_{\rm atm} = 2.5 \times 10^{-3}\,{\rm eV}^2, \quad \sin^2 2\theta_{\rm atm} = 1. \qquad (1)$$

Among the four oscillation solutions for the solar neutrino problem, the large mixing angle MSW (LMA) solution is most favored, followed by the LOW and VAC solutions [5–7]. The best fit values for the LMA solution are [7]

$$\Delta m^2_{\rm sol} = 5 \times 10^{-5}\,{\rm eV}^2, \quad \tan^2\theta_{\rm sol} = 0.42. \qquad (2)$$

The same analysis excludes the small mixing angle (SMA) solution at the $3.7\sigma$ level.

On the theoretical side, hundreds of neutrino mass models have been constructed in the literature [8], each trying to explain to a greater or lesser degree the two afore-mentioned features. A consensus has now emerged that the see-saw mechanism seems to be the most natural and economical way to account for the tiny neutrino masses.

In the see-saw mechanism, the standard model (SM) is extended by including the right-handed Majorana neutrinos, $\nu_{\rm R}$. Since $\nu_{\rm R}$ are the SM gauge group, $SU(2)_W \times U(1)_Y$, singlets, their masses are not protected by the SM gauge symmetry. The $\nu_{\rm R}$ may get masses at a very high energy scale and may be much heavier than the SM particles. Having both left- and right-handed neutrinos and the $\nu_{\rm R}$ being singlets, the neutrinos can have both Dirac mass terms,

$$\mathcal{L}_{\rm D} = -M_{\rm D}\bar{\nu}_{\rm L}\nu_{\rm R} + {\rm h.c.}, \qquad (3)$$

and Majorana mass terms,

$$\mathcal{L}_{\rm M} = -\frac{1}{2}M_{\rm R}\nu_{\rm R}^{\rm T}C\nu_{\rm R} + {\rm h.c.}, \qquad (4)$$

with $C$ being the charge conjugate matrix. Integrating out the heavy right-handed neutrinos, we get the Majorana mass terms for the left-handed neutrinos,

$$\mathcal{L}_\nu = -\frac{1}{2}M_\nu\nu_{\rm L}^{\rm T}C\nu_{\rm L} + {\rm h.c.}, \qquad (5)$$

with

$$M_\nu = -M_{\rm D}M_{\rm R}^{-1}M_{\rm D}^{\rm T}. \qquad (6)$$

Since $M_{\rm R} \gg M_{\rm D} \sim M_{\rm EW}$, we have that $M_\nu$ is much smaller than the electro-weak scale $M_{\rm EW}$.

The see-saw mechanism is typically realized within the framework of a supersymmetric (SUSY) grand unified theory (GUT), which adds further desirable features including unification of the SM gauge couplings at the GUT scale and avoidance of the SM hierarchy problem. In an SO(10) GUT, the see-saw mechanism is a natural outcome of the group theory.

However, no generally accepted mechanism has yet been put forth to explain the large neutrino mixing angles until now [8]. The difficulty relies on the two facts that

(i) the neutrino spectrum exhibits a large hierarchy, which usually means small mixing among the neutrinos, and

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(ii) in grand unified models the lepton and the quark mass matrices are closely related, which generally makes it difficult to accommodate small quark mixing and large lepton mixing in one scheme.

An elegant idea proposed to explain the large neutrino mixing angle is the so called "lopsided" structure [9,10]. In this scheme the neutrino mass matrix, $M_\nu$, produces small mixing according to (i). However, the charged lepton mass matrix, $M_{\rm L}$, produces large mixing and the difficulty relying on (ii) is cleverly solved. As we know, the neutrino mixing is actually the mismatch between $M_{\rm L}$ and $M_\nu$. Diagonalizing $M_{\rm L}$ and $M_\nu$ by

$$U_{\rm L}^\dagger M_{\rm L} U_{\rm R} = {\rm diag}(m_e, m_\mu, m_\tau), \quad (7)$$

and

$$U_\nu^\dagger M_\nu U_\nu = {\rm diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \quad (8)$$

we have the neutrino mixing matrix

$$V_{\rm MNS} = U_{\rm L}^\dagger U_\nu. \quad (9)$$

So the large mixing in $U_{\rm L}$ leads to large mixing in the physical mixing matrix, $V_{\rm MNS}$.

The "lopsided" structure works as follows. In an SU(5) grand unified model, the left-handed charged leptons are in the same multiplets as the $CP$ conjugates of the right-handed down-type quarks, and therefore $M_{\rm L}$ is closely related to the *transpose* of the mass matrix of the down-type quarks, $M_{\rm down}$. The two mass matrices have the following approximate forms:

$$M_{\rm L} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \sigma \\ 0 & \epsilon & 1 \end{pmatrix} m_{\rm D}$$

and

$$M_{\rm down} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \sigma & 1 \end{pmatrix} m_{\rm D}, \quad (10)$$

respectively, with $\sigma \sim 1$, $\epsilon \ll 1$, and the zeros representing entries much smaller than $\epsilon$. For $M_{\rm L}$, $\sigma$ controls the mixing between the second and the third families of the left-handed leptons[1], which greatly enhances $\theta_{\rm atm}$, while $\epsilon$ controls the mixing between the second and the third families of the right-handed leptons, which is not observable at low energy. For the quarks the roles of $\sigma$ and $\epsilon$ are reversed: the small $\mathcal{O}(\epsilon)$ mixing is in the left-handed sector, accounting for the smallness of $V_{cb}$, while the large $\mathcal{O}(\sigma)$ mixing is in the right-handed sector, which is not observable.

A larger gauge group with SU(5) being its subgroup also has the above property. Many realistic supersymmetric grand unified models have been built based on the ideas of the see-saw mechanism and "lopsided" structure in the literature to account for the neutrino properties [9,10]. All such models have a definite prediction – the lepton flavor violation (LFV) at low energy, which can be used to test this kind of models. We investigate the LFV prediction in this kind of models.

[1] Here we use the convention that a left-handed doublet multiplies the Yukawa coupling matrix from the left side while a right-handed singlet multiplies the matrix from the right side

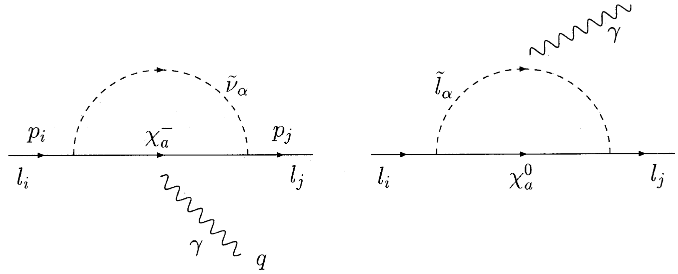


**Fig. 1.** Feynman diagrams for the process $l_i \to l_j\gamma$ via the exchange of a chargino (*left*) and via a neutralino (*right*)

## 2 Lepton flavor violation in supersymmetry

In a supersymmetric model, the soft SUSY-breaking terms may induce large lepton flavor violation. The possible LFV sources are the off-diagonal terms of the slepton mass matrices $(m_{\tilde{\rm L}}^2)_{ij}$, $(m_{\tilde{\rm R}}^2)_{ij}$ and the trilinear couplings $A_{ij}^{\rm L}$. The present experimental bounds on the LFV processes give strong constraints on such off-diagonal terms, with the strongest constraint coming from ${\rm Br}(\mu \to e\gamma)$ ($< 1.2\times10^{-11}$ [11]). We have to find a mechanism to align the lepton and the scalar lepton bases. This is the so called SUSY flavor problem.

A generally adopted way to avoid these dangerous off-diagonal terms is to impose universality constraints on the soft terms at the SUSY-breaking scale, such as in the gravity-mediated [12] or gauge-mediated [13] SUSY-breaking scenarios. Yet, even with the universality condition, off-diagonal terms can be induced at lower energy scales through quantum effects. Such LFV effects induced in the SUSY see-saw mechanism are given in the next section. We first give the general analytic expressions for the branching ratios of the LFV processes, $l_i \to l_j\gamma$.

The LFV decay, $l_i \to l_j\gamma$, occurs through the photon-penguin diagrams shown in Fig. 1. The amplitude for the processes takes the general form

$$M = e m_i \bar{u}_j(p_j) {\rm i}\sigma_{\mu\nu} q^\nu (A_{\rm L}^{ij} P_{\rm L} + A_{\rm R}^{ij} P_{\rm R}) u_i(p_i) \epsilon^\mu(q). \quad (11)$$

The contribution from neutralino exchange gives

$$A_{\rm L}^{(n)} = -\frac{1}{32\pi^2}\left(\frac{e}{\sqrt{2}\cos\theta_{\rm W}}\right)^2 \frac{1}{m_{\tilde{l}_\alpha}^2}\left[B^{j\alpha a *} B^{i\alpha a} F_1(k_{\alpha a}) + \frac{m_{\chi_a^0}}{m_i} B^{j\alpha a *} A^{i\alpha a} F_2(k_{\alpha a})\right], \quad (12)$$

$$A_{\rm R}^{(n)} = A_{\rm L}^{(n)}(B \leftrightarrow A), \quad (13)$$

where

$$F_1(k) = \frac{1 - 6k + 3k^2 + 2k^3 - 6k^2\log k}{6(1-k)^4}, \quad (14)$$

$$F_2(k) = \frac{1-k^2+2k\log k}{(1-k)^3}, \tag{15}$$

with $k_{\alpha a} = m^2_{\chi^0_a}/m^2_{\tilde{l}_\alpha}$. $A$ and $B$ are the lepton–slepton–neutralino coupling vertices given by

$$A^{i\alpha a} = \left( Z^{i\alpha}_{\tilde{\mathrm{L}}}(Z^{1a}_{\mathrm{N}} + Z^{2a}_{\mathrm{N}}\cot\theta_{\mathrm{W}}) - \cot\theta_{\mathrm{W}} \frac{m_i}{M_W\cos\beta} Z^{(i+3)\alpha}_{\tilde{\mathrm{L}}} Z^{3a}_{\mathrm{N}} \right), \tag{16}$$

$$B^{i\alpha a} = -\left( 2Z^{(i+3)\alpha}_{\tilde{\mathrm{L}}} Z^{1a*}_{\mathrm{N}} + \cot\theta_{\mathrm{W}} \frac{m_i}{M_W\cos\beta} Z^{i\alpha}_{\tilde{\mathrm{L}}} Z^{3a*}_{\mathrm{N}} \right), \tag{17}$$

where $Z_{\tilde{\mathrm{L}}}$ is the $6\times 6$ slepton mixing matrix and $Z_{\mathrm{N}}$ is the neutralino mixing matrix. The corresponding contribution coming from chargino exchange is

$$A^{(c)}_{\mathrm{L}} = \frac{g_2^2}{32\pi^2} Z^{i\alpha*}_{\tilde{\nu}} Z^{j\alpha}_{\tilde{\nu}} \frac{1}{m^2_{\tilde{\nu}_\alpha}} \left[ Z^-_{2a} Z^{-\,*}_{2a} \frac{m_i m_j}{2M_W^2\cos^2\beta} F_3(k_{\alpha a}) + \frac{m_{\chi^-_a}}{\sqrt{2}M_W\cos\beta} Z^+_{1a} Z^-_{2a} \frac{m_j}{m_i} F_4(k_{\alpha a}) \right], \tag{18}$$

$$A^{(c)}_{\mathrm{R}} = \frac{g_2^2}{32\pi^2} Z^{i\alpha*}_{\tilde{\nu}} Z^{j\alpha}_{\tilde{\nu}} \frac{1}{m^2_{\tilde{\nu}_\alpha}} \left[ Z^+_{1a} Z^{+\,*}_{1a} F_3(k_{\alpha a}) + \frac{m_{\chi^-_a}}{\sqrt{2}M_W\cos\beta} Z^{+\,*}_{1a} Z^{-\,*}_{2a} F_4(k_{\alpha a}) \right], \tag{19}$$

where

$$F_3(k) = \frac{2+3k-6k^2+k^3+6k\log k}{6(1-k)^4}, \tag{20}$$

$$F_4(k) = \frac{3-4k+k^2+2\log k}{(1-k)^3}, \tag{21}$$

with $k_{\alpha a} = m^2_{\chi^-_a}/m^2_{\tilde{\nu}_\alpha}$. $Z_{\tilde{\nu}}$ is the sneutrino mixing matrix, while $Z^+$ and $Z^-$ are the chargino mixing matrices.

The branching ratio for $l_i \to l_j\gamma$ is given by

$$\mathrm{Br}(l_i \to l_j\gamma) = \frac{\alpha_{\mathrm{em}}}{4} m_i^5 (|A^{ij}_{\mathrm{L}}|^2 + |A^{ij}_{\mathrm{R}}|^2)/\Gamma_i, \tag{22}$$

where $\Gamma_i$ is the width of $l_i$. To identify the parameter dependence one may use the mass insertion approximation [14], which yields, for large $\tan\beta$,

$$\mathrm{Br}(l_i \to l_j\gamma) \sim \frac{\alpha^3}{G_F^2} \frac{[(m^2_{\tilde{\mathrm{L}}})_{ij}]^2}{m_s^8} \tan^2\beta, \tag{23}$$

where $m_s$ represents the common slepton mass. We can see that the supersymmetric contribution to $\mathrm{Br}(l_i \to l_j\gamma)$ is proportional to $\tan^2\beta$ and to the amount of the off-diagonal terms in the slepton mass matrix.

## 3 Radiatively produced LFV in the see-saw mechanism

Although the soft terms are universal at the GUT (or Planck) scale, off-diagonal soft terms may be radiatively produced in the see-saw mechanism. Especially, if the charged lepton mass matrix is "lopsided", the radiatively produced LFV effects are large enough to be observed. We will show this below.

At the energy scales between $M_{\mathrm{R}}$ and $M_{\mathrm{GUT}}$, the superpotential of the lepton sector is given by

$$W = Y^{ij}_{\mathrm{N}} \hat{H}_2 \hat{L}_i \hat{N}_j + Y^{ij}_{\mathrm{L}} \hat{H}_1 \hat{L}_i \hat{E}_j + \frac{1}{2} M^{ij}_{\mathrm{R}} \hat{N}_i \hat{N}_j + \mu \hat{H}_1 \hat{H}_2, \tag{24}$$

where $Y_{\mathrm{N}}$ and $Y_{\mathrm{L}}$ are the neutrino and charged lepton Yukawa coupling matrices, respectively. In general, $Y_{\mathrm{N}}$ and $Y_{\mathrm{L}}$ cannot be diagonalized simultaneously. This bases misalignment can lead to lepton flavor violation, similar to the quark sector. This LFV effects can transfer to the soft terms through quantum effects and induce non-diagonal terms below the GUT scale. This is clearly shown by the following renormalization group equation (RGE) for $m^2_{\tilde{\mathrm{L}}}$, which gives the dominant contribution to low energy LFV processes:

$$\mu\frac{\mathrm{d}m^2_{\tilde{\mathrm{L}}}}{\mathrm{d}\mu} = \frac{2}{16\pi^2} \left[ -\Sigma c_i g_i^2 M_i^2 + \frac{1}{2}\left[ Y_{\mathrm{N}} Y^\dagger_{\mathrm{N}} m^2_{\tilde{\mathrm{L}}} + m^2_{\tilde{\mathrm{L}}} Y_{\mathrm{N}} Y^\dagger_{\mathrm{N}} \right] + \frac{1}{2}\left[ Y_{\mathrm{L}} Y^\dagger_{\mathrm{L}} m^2_{\tilde{\mathrm{L}}} + m^2_{\tilde{\mathrm{L}}} Y_{\mathrm{L}} Y^\dagger_{\mathrm{L}} \right] + Y_{\mathrm{L}} m^2_{\tilde{E}} Y^\dagger_{\mathrm{L}} + m^2_{H_{\mathrm{D}}} Y_{\mathrm{L}} Y^\dagger_{\mathrm{L}} + E_A E^\dagger_A + Y_{\mathrm{N}} m^2_{\tilde{N}} Y^\dagger_{\mathrm{N}} + m^2_{H_U} Y_{\mathrm{N}} Y^\dagger_{\mathrm{N}} + N_A N^\dagger_A \right]. \tag{25}$$

Here $E_A = A^{\mathrm{L}} \cdot Y_{\mathrm{L}}$ and $N_A = A^{\mathrm{N}} \cdot Y_{\mathrm{N}}$, while $g_i$ and $M_i$ are the gauge coupling constants and gaugino masses, respectively.

$Y_{\mathrm{L}}$ and $Y_{\mathrm{N}}$ can be diagonalized by bi-unitary rotations

$$Y^\delta_{\mathrm{L}} = U^\dagger_{\mathrm{L}} Y_{\mathrm{L}} U_{\mathrm{R}}, \quad Y^\delta_{\mathrm{N}} = V^\dagger_{\mathrm{L}} Y_{\mathrm{N}} V_{\mathrm{R}}, \tag{26}$$

respectively. Lepton flavor mixing is determined by the matrix $V_{\mathrm{D}}$, the analog to $V_{\mathrm{KM}}$ in the quark sector, defined by

$$V_{\mathrm{D}} = U^\dagger_{\mathrm{L}} V_{\mathrm{L}}. \tag{27}$$

We see that $V_{\mathrm{D}}$ only exists above the energy scale $M_{\mathrm{R}}$. It is different from the MNS matrix $V_{\mathrm{MNS}}$ in (9).

Then running the RGEs between $M_{\mathrm{GUT}}$ (where the initial soft terms are universal) and $M_{\mathrm{R}}$ (where $\nu_{\mathrm{R}}$ decouples and no LFV interactions are below) leads to the flavor mixing off-diagonal terms. In the basis where $Y_{\mathrm{L}}$ is diagonal, the off-diagonal terms of $m^2_{\tilde{\mathrm{L}}}$ can be approximately given by

$$\left(\delta m^2_{\tilde{\mathrm{L}}}\right)_{ij} \approx \frac{1}{8\pi^2} (Y_{\mathrm{N}} Y^\dagger_{\mathrm{N}})_{ij} (3+a^2) m_0^2 \log\frac{M_{\mathrm{GUT}}}{M_{\mathrm{R}}} \approx \frac{1}{8\pi^2} (V_{\mathrm{D}})_{i3} (V^*_{\mathrm{D}})_{j3} Y^2_{N_3} (3+a^2) m_0^2 \log\frac{M_{\mathrm{GUT}}}{M_{\mathrm{R}}}, \tag{28}$$

where, assuming the three generations' Yukawa couplings in $Y_{\mathrm{N}}$ are hierarchical, only the third generation's Yukawa coupling, $Y_{N_3}$, is retained. The "$a$" is the universal trilinear coupling given by $A_0 = am_0$, and $m_0$ is the universal slepton mass at $M_{\mathrm{GUT}}$.

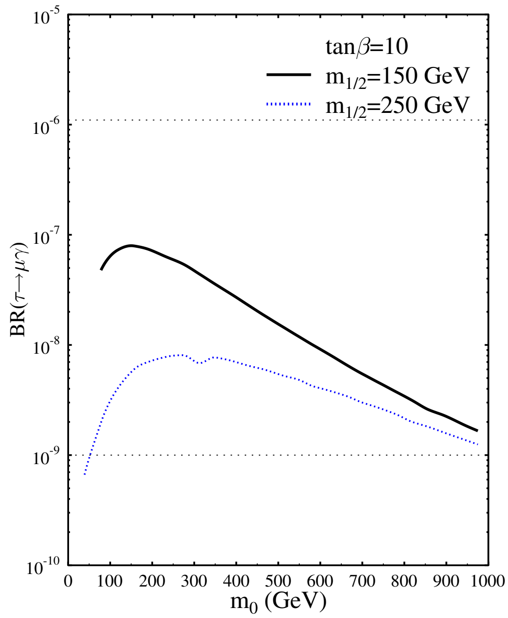


**Fig. 2.** Branching ratio of $\tau \to \mu\gamma$ as a function of $m_0$ for $A_0 = m_0$, $\tan\beta = 10$ and $m_{1/2} = 150\,\mathrm{GeV}, 250\,\mathrm{GeV}$. The dotted lines are the present upper bound and the expected sensitivity

Equation (28) clearly shows that the mixing matrix $V_{\mathrm{D}}$ determines $\delta m^2_{\mathrm{L}}$. The "lopsided" models predict a large mixing in $U_{\mathrm{L}}$, and therefore a large mixing in $V_{\mathrm{D}}$, which finally leads to observable LFV effects.

## 4 Numerical results

The precise results are obtained by solving the coupled RGEs numerically. The RGEs below $M_{\mathrm{R}}$ are the set of equations for MSSM, while above $M_{\mathrm{R}}$ the equations must be extended by including $\nu_{\mathrm{R}}$ and corresponding scalar partners. The details for solving the equations are given in [15].

For the process $\tau \to \mu\gamma$ we note that its branching ratio is approximately proportional to $|(V_{\mathrm{D}})_{23}(V_{\mathrm{D}})_{33}|^2$. This quantity is quite model independent since all the "lopsided" models give a large, near maximal, 2–3 mixing. Thus we can give a quite definite prediction for this process.

$\mathrm{Br}(\tau \to \mu\gamma)$ is plotted in Fig. 2 for a typical set of SUSY parameters. We notice that in a quite large parameter space the process $\tau \to \mu\gamma$, induced in the supersymmetric see-saw mechanism, is below the present experimental bound, $1.1 \times 10^{-6}$ [16], while it will be detected in the future experiment if the expected sensitivity can reach down to $10^{-9}$ [17]. In our calculation the SUSY parameters are constrained by the $g_\mu - 2$ anomaly [18], so $m_{1/2}$ cannot be too large.

The branching ratio of $\mu \to e\gamma$ is approximately proportional to $|(V_{\mathrm{D}})_{13}(V_{\mathrm{D}})_{23}|^2$. The element $(V_{\mathrm{D}})_{13}$ seems to be quite model dependent. However, under the following observations and assumptions, we find that a general prediction of $(V_{\mathrm{D}})_{13}$ in this kind of models is possible [19].

First, we assume that $Y_{\mathrm{N}}$ has a similar hierarchical structure as the Yukawa coupling matrix of the up-type quark, $Y_u$. In SO(10) grand unified models, the simplest symmetry breaking mechanism leads to $Y_{\mathrm{N}} = Y_u$. Since the see-saw mechanism is usually realized in an SO(10) grand unified model, this assumption is quite general. $Y_u$ is constrained by the values of the up-type quark masses and the CKM mixing angles. By our second assumption that there is no accidental cancellation between the mixing matrices for the up- and down-type quarks leading to small CKM mixing, we then have

$$\theta_u^{13} \lesssim V_{td} \sim 0.008, \tag{29}$$

with $\theta_u^{13}$ and $V_{td}$ being the 1–3 mixing angle produced by $Y_u$ and the 3–1 element of the CKM matrix. We thus expect that the 1–3 mixing angle produced by $Y_{\mathrm{N}}$, $\theta_{\mathrm{N}}^{13}$, is of the same order of magnitude as $\theta_u^{13}$. Then we have $\theta_{\mathrm{N}}^{13} \lesssim 0.008$. Analogously, we have $\sin\theta_{\mathrm{N}}^{23} \lesssim V_{ts} \cong 0.04$. Third, we observed that in most models $m_\tau$ and $m_\mu$ got their masses mainly from the 2–3 block of the lepton mass matrix. The elements in the first row and the first column of $M_{\mathrm{L}}$ are constrained by $m_e$. By this structure, as given in (10), one finds that [8,20]

$$\sin\theta_{12} \sim \sqrt{m_e/m_\mu} \cong 0.07, \tag{30}$$

and

$$\sin\theta_{13} \approx m_\mu/m_\tau \sin\theta_{12} \ll \sin\theta_{12}, \tag{31}$$

with $\theta$ being the mixing angles in $U_{\mathrm{L}}$. Finally, taking into account that $\theta_{23} \sim \mathcal{O}(1)$ in "lopsided" models, we get

$$(V_{\mathrm{D}})_{13} \approx \sin\theta_{12}\sin\theta_{23} \approx 0.05, \tag{32}$$
$$(V_{\mathrm{D}})_{23} \approx -\sin\theta_{23} \approx -0.71. \tag{33}$$

Thus the angles in $U_{\mathrm{L}}$ alone can determine $(V_{\mathrm{D}})_{13}$ and $(V_{\mathrm{D}})_{23}$. This conclusion certainly depends on the assumed forms of $Y_{\mathrm{L}}$ and $Y_{\mathrm{N}}$; nonetheless, it is correct in most published "lopsided" models [9,10], which can be explicitly checked. In fact our assumptions are implied in these models.

Actually the above assumptions can be relaxed. Since (32) is one term, the dominant one here, of the full expression for $(V_{\mathrm{D}})_{13}$, unless there is strong cancellation among these terms, we always have $(V_{\mathrm{D}})_{13}$ being $\mathcal{O}(0.05)$ or larger.

In Fig. 3 we give our numerical result for $\mathrm{Br}(\mu \to e\gamma)$. Taking $\tan\beta = 10$ and $\theta_{12} = 0.07$ as the typical value of the mixing angle between the first and the second generations in $U_{\mathrm{L}}$, we find that the predicted $\mathrm{Br}(\mu \to e\gamma)$ has already exceeded the present upper bound, $1.2 \times 10^{-11}$ [11]. The other set of curves are for $\theta_{12} = 0.01$ (corresponding to $(V_{\mathrm{D}})_{13} = 0.007$). In this case $\mathrm{Br}(\mu \to e\gamma)$ may be below the experimental limit.

So $\mathrm{Br}(\mu \to e\gamma)$ is large because of the large mixing angle $\theta_{23}$, which features the "lopsided" model and gives a satisfying solution to the large neutrino mixing. However,

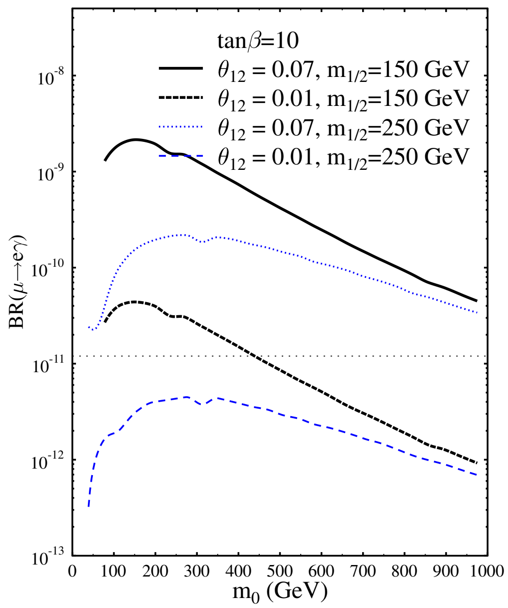


**Fig. 3.** $\mathrm{Br}(\mu \to e\gamma)$ as a function of $m_0$ for $\tan\beta = 10$ and $m_{1/2} = 150\,\mathrm{GeV}, 250\,\mathrm{GeV}$. $A_0 = m_0$ and $\mu > 0$ are assumed. $\theta_{12}$ is the mixing angle between the first and second generations in $U_\mathrm{L}$. The horizontal dotted line is the present experimental limit, $1.2 \times 10^{-11}$ [11]

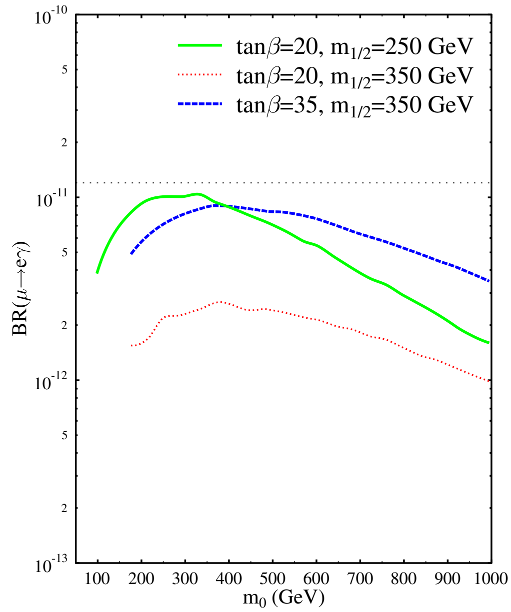


**Fig. 4.** $\mathrm{Br}(\mu \to e\gamma)$ predicted by our model as a function of $m_0$ for $\tan\beta = 20$, $m_{1/2} = 250\,\mathrm{GeV}, 350\,\mathrm{GeV}$ and $\tan\beta = 35$, $m_{1/2} = 350\,\mathrm{GeV}$. The horizontal dotted line is the present experimental limit, $1.2 \times 10^{-11}$ [11]

a large $\theta_{23}$ enhances both $(V_\mathrm{D})_{13}$ and $(V_\mathrm{D})_{23}$, as given in (32) and (33), leading to a too large $\mathrm{Br}(\mu \to e\gamma)$. This is really a dilemma. Another shortcoming of the "lopsided" model is that it generally predicts a SMA or VAC solution to the solar neutrino problem, which are disfavored by the present data. A recent work in [9] predicts the LMA solution by the "lopsided" structure. However, fine tuning to some extent is needed in this model. In the next section we propose a new structure for $M_\mathrm{L}$, which can solve the above problems simultaneously. This structure predicts a very small $(V_\mathrm{D})_{13}$ while, at the same time, it yields the LMA solution to the solar neutrinos.

## 5 A new neutrino mass model

Assuming $Y_\mathrm{N}$ is nearly diagonal we have

$$M_\mathrm{L} = \begin{pmatrix} 0 & \delta & \sigma \\ -\delta & 0 & 1-\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} m,$$

with

$$\sigma \sim \mathcal{O}(1), \quad \delta \ll \epsilon \ll 1. \tag{34}$$

Taking

$$\delta = 0.00077, \quad \epsilon = 0.12 \quad \text{and} \quad \sigma = 0.58, \tag{35}$$

we can obtain the correct mass ratios $m_e/m_\mu$, $m_\mu/m_\tau$ and predict the neutrino mixing parameters to be

$$\sin^2 2\theta_\mathrm{atm} = 0.998,$$

$$\tan^2\theta_\mathrm{sol} = 0.42$$
$$\text{and} \quad U_{e3} = -0.0054. \tag{36}$$

The notable feature of (34) compared with the usual "lopsided" models is the $\mathcal{O}(1)$ element $\sigma$. Both the $(2,3)$ and $(1,3)$ elements in $M_\mathrm{L}$ are large, naturally leading to large mixing angles, $\theta_{23}$ and $\theta_{12}$.

The prediction of $U_{e3} = -0.0054$ is non-trivial, since the three parameters are fixed by the lepton mass ratios and one neutrino mixing angle. It thus provides a test of our model.

Diagonalizing $M_\mathrm{L}$ analytically we can express $U_{e3}$ by

$$U_{e3} \cong \frac{m_e}{m_\mu} \cdot U_{\mu 3}/\tan\theta_\mathrm{sol}. \tag{37}$$

This prediction, that $U_{e3}$ is proportional to $m_e/m_\mu$, is unique. Usually $U_{e3}$ is predicted to be proportional to $(m_e/m_\mu)^{1/2}$. Our model gives a very small $U_{e3}$ value. Another interesting example which also gives a quite small $U_{e3}$ is in [21], which predicts

$$U_{e3} = \sqrt{\frac{m_e}{m_\tau}} U_{\mu 3}.$$

However, this model predicts $\theta_\mathrm{sol} \approx \pi/4$, which is excluded by the present data.

The prediction of $\mathrm{Br}(\mu \to e\gamma)$ by our model is plotted in Fig. 4. In most of the parameter space our model predicts $\mathrm{Br}(\mu \to e\gamma)$ to be below the present experimental limit, while being large enough to be detected in the next generation experiment [22].

## 6 Summary and conclusions

A quite popular theoretical scheme to explain the atmospheric and solar neutrino experiments is the see-saw mechanism together with the "lopsided" charged lepton mass matrix. This scheme is generally realized in the framework of supersymmetric grand unification. Our analysis shows that such a structure may predict a large lepton flavor violation at low energy. The process $\tau \to \mu\gamma$ is quite promising to test whether there is a large mixing in the charged lepton sector, as predicted by "lopsided" models. In most SUSY parameter space this process will be detected in the next generation experiment. The "lopsided" models also make a model-insensitive prediction for the process $\mu \to e\gamma$. However, the branching ratio of $\mu \to e\gamma$ predicted by these models generally exceeds the present experimental limit. An extended "lopsided" form of the charged lepton mass matrix is then proposed to solve this problem. The new structure can produce maximal 2–3 mixing, large 1–2 mixing, while very small 1–3 mixing is predicted in the lepton sector. $\mathrm{Br}(\mu \to e\gamma)$ is thus suppressed below the present experimental limit. The LMA solution for the solar neutrino problem is naturally produced.

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